1. (10%) Let A, B be sets. Use set identities (basic laws of set theory) to prove that 
\((A - B) \cup (B - A) = (A \cup B) - (A \cap B)\)

2. (10%) Prove that if \(x^3 - 5x^2 + 3x + 4 = 0\), then \(x \neq 1\).

3. (10%) Use mathematical induction to prove that for any positive integer \(n > 1\),
\[
\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} > \sqrt{n}
\]

4. (20%) This question concerns the recursive relations.
(a) Write a recursive definition for the sequence \(\{ a_n \}\), where \(a_n = a^n \) for \(n \geq 1\)
(b) Consider another sequence \(\{ b_n \}\), whose recursive definition is as follows:
\(b_1 = 0, b_2 = 2, \) and \(b_n = 3b_{n-1} - 2b_{n-2}\) for \(n \geq 3\). Now, find a formula (in terms of \(n\) only) for \(b_n\).

5. (10%) Are the following two graphs isomorphic? Why?

6. (20%) In a string \(S\), a subsequence is obtained by removing some characters out of \(S\). For example, TTA is a subsequence of AGTCATA, but GGA is not.
(a) Now, given any two strings, string \(s\) of length \(m\) and string \(t\) of length \(n\), device an algorithm (in pseudocode) to check whether \(t\) is a subsequence of \(s\).
(b) Analyze the worse-case time complexity of your algorithm in (a).

7. (20%) Let \((S, R)\) be a partially ordered set. \(S\) is a set of strings
\{ TA, GC, GG, TGAC, TGGA, GAGTC, GTGT, GTGGTA, GAGTCGT \} and \(R\) is the partial order relation defined on \(S\) where \(xRy\) means \(x\) is a subsequence of \(y\).
(a) Draw the Hasse diagram for \(R\).
(b) Find all maximal elements.
(c) Find lub( \{ GC, GC \} )
(d) Find glb( \{ GTGGTA, GAGTCGT \} )
(e) Is this partially ordered set a lattice? Why?