1. (10%) Find \( \int_{0}^{2} \frac{1}{x-1} \, dx \)

2. (15%) Find \( \int_{1}^{e} \frac{e^x}{1-e^{2x}} \, dx \)

3. (15%) The region enclosed by the curves \( y = x \) and \( y = \sqrt{x} \) is rotated about the line \( x = 1 \). Find the volume of the resulting solid.

4. (15%) Find \( \iint_{R} \frac{2y}{x^2 + y^2} \, dA \), where \( R \) is the region bound by the lines \( y = x \), \( y = 2x \) and \( x = 1 \).

5. (15%) Find the limit, if it exists, or show that the limit does not exist.
   (a) \( \lim_{(x,y) \to (1,2)} (x^2 + 4xy - 5y^3) \)
   (b) \( \lim_{(x,y) \to (0,0)} \frac{y^3}{x^2 + 2y^3} \)
   (c) \( \lim_{(x,y) \to (0,0)} \frac{xy^2}{x^2 + y^4} \)

6. (15%) Determine whether the series is convergent or divergent. If it is convergent, find its sum.
   (a) \( \sum_{n=2}^{\infty} \frac{1}{n(n-1)} \)
   (b) \( \sum_{n=0}^{\infty} \frac{2^n}{n!} \)
   (c) \( \sum_{n=1}^{\infty} \frac{2n^3}{n^3 + 1} \)

7. (15%) Find the Maclaurin series for \( f(x) = e^{2x} \).

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