

銘傳大學 95 學年度轉學生招生考試

資訊管理學系、資訊傳播工程學系、資訊工程學系、
電腦與通訊工程學系、醫療資訊與管理學系

7月25日第四節

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微積分試題

(限用答案本作答)

(1) Find each limit: (10 %)

$$(a) \lim_{x \rightarrow 2} \frac{x-2}{x^2-x-2}$$

$$(b) \lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2}$$

(2) Find the derivative $\frac{dy}{dx}$: (20 %)

$$(a) y = x^2 + 1$$

$$(b) y = \sqrt{x^2 + 1}$$

$$(c) y = \ln(x + \sqrt{x^2 + 1})$$

$$(d) y = \frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)$$

(3) Find the equation of the tangent to the curve

$y = x^3 - 3x^2 + 1$ at the point of inflection. (10 %)

(4) Find the relative maximum and minimum values for the function $f(x) = \sqrt[3]{x^2(x-3)}$. (10 %)

(5) Find the average value of $f(x) = \frac{xe^x}{(1+x)^2}$ over $[0, 2]$. (10 %)

(6) Find the area bounded by $y = x^2$ and $y = 2x - x^2$. (10 %)

(7) Evaluate: (10 %)

$$(a) \int_0^2 \int_0^1 4xy \, dx \, dy$$

$$(b) \int_0^1 \int_y^1 e^{x^2} \, dx \, dy$$

(8) Let $u = \ln\sqrt{x^2 + y^2}$, show that $u_{xx} + u_{yy} = 0$. (10 %)

(9) Let $f(x, y) = \int_y^{x^2} \frac{\sin t}{\sqrt{1+t^2}} dt$, find $f_x(0, \frac{\pi}{2})$ and $f_y(0, \frac{\pi}{2})$. (10 %)