

銘傳大學九十學年度轉學生招生考試

應統 轉三

七月三十日 第四節

應用機率論 試題

*可以使用計算機

1. 試述中央極限定理〈The Central Limit Theorem, C.L.T.〉(10%)
2. 設 $X \sim \text{Poisson}(\lambda)$ ，試求其動差母函數〈the moment generating function, MGF〉(10%)
3. 袋中有 3 黑球、2 白球。今任意取出 3 球，問得 2 黑球 1 白球的機率為何？
 - a) 每次皆不放回
 - b) 每次皆放回
4. 設 X 、 Y 的 joint density function 為 $f(x, y) = e^{-\frac{x}{y}} e^{-\frac{y}{y}}$ $0 < x < \infty$ ， $0 < y < \infty$ ； $f(x, y) = 0$ otherwise。求 $E[X|Y=y]$ (10%)
5. A laboratory blood test is 0.95 effective in detecting a certain disease when it is, in fact, present. However, if a healthy person is tested, with probability 0.01 the test will imply this person has the disease. Suppose 0.005 of the population has the disease, what is the probability a person has the disease given that the test result implies this person has the disease given that the test result implies this person has the disease? (Bayes' Theorem)(10%)
6. 設 X 的 density function 為 $f = 3e^{-3x}$ $0 < x$ ； $f(x) = 0$ otherwise。求 $P(2 < X < 7)$ (10%)
7. 設 $X \sim N(0,1)$ ，即 X 的 density function 為 $f(x, y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ ， $-\infty < x < \infty$ ，求的 $Y = X^2$ density function, $h(y)$ 。(10%)
8. Suppose X is uniformly distributed over the interval $(0, 1)$. The density function of X is $f(x) = 1$ of $0 < x < 1$ ； $f(x) = 0$ otherwise. Compute $E[X]$ and $\text{Var}(X)$ (10%)
9. 設 X_1 、 X_2 、 X_3 為 i. i. d. (independently identically distributed) random variables. 設

X_i 的 density function $f(x) = \lambda e^{-\lambda x}$ $0 < x < \infty$; $f(x) = 0$ otherwise, ($\lambda > 0$)。求 $Y = X_1 + X_2 + X_3$ 的 density function。(10%)

〈 試題完 〉