統計學(二) 試題

可攜帶簡單型計算器。依題號次序在答案本上作答，顛倒次序不予評分。
統計檢定假設問題，一律以α=0.05 作答。

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with μ and σ^2
   a) Prove that the sample mean, $\overline{X}$, is an unbiased estimator of μ. (8%)
   b) Show that the variance of sample mean is σ^2/n. (6%)
   c) Find $E(\overline{X}^2)$. (6%)

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with μ and σ^2.
   a) Derive the $(1-\alpha)100\%$ confidence interval for μ, given σ^2 known. (10%)
   b) Suppose that σ^2=100 and n=25 with a sample mean of 12. Use a) to construct a 95% confidence interval for μ. (10%)

3. Suppose that a random variable X has a probability density function of
$$f(x) = \begin{cases} 2e^{-2x} & x>0 \\ 0 & \text{elsewhere} \end{cases}$$
   a) Find $P(X > 2)$. (10%)
   b) Find $P(X > 3 \mid X > 2)$. (10%)

4. a) Describe the Central Limit Theorem. (10%)
   b) Let the population be χ^2 with the parameter of 1. If a random sample of size 50 is drawn, then find the probability that the sample mean is greater than 1.3. (10%)

5. Multiple regression is used by accountants in cost analysis to shed light on the factors that cause costs to be incurred and the magnitudes of their effects. Suppose that an account wish to study the linear relationship between the number of labor hours (y) required and three factors (x1=pounds shipped per 1,000; x2=percentage of units shipped by truck; x3=average shipment weight in lbs.), which are considered having effects on y. The sample data were collected and SAS computer output is shown below. (4% each)
【取用 SAS 報表中數字時，保留小數四位，第五位小數四捨五入。】

本試題係兩面印刷

統計學(二) 試題

Dependent Variable: Y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	5158.31362780	1719.43794260	17.87	0.0001
Error	16	1539.88617220	96.24288576		
C. Total	19	6698.20000000			

R-Square	C.V.	Root MSE	Y Mean
0.770104	10.51484	9.81034585	93.30000000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
X1	1	3400.64083349	3400.64083349	35.33	0.0001
X2	1	198.35671076	198.35671076	2.06	0.1704
X3	1	1559.31628354	1559.31628354	16.20	0.0010

Source	DF	Type III SS	Mean Square	F Value	Pr > F
X1	1	138.19243705	138.19243705	1.44	0.2483
X2	1	24.62499570	24.62499570	0.26	0.6199
X3	1	1559.31628354	1559.31628354	16.20	0.0010

Parameter	Estimate	T for H0: Parameter=0	Pr > |T|	Std Error of Estimate
INTERCEPT	131.9242521	5.13	0.0001	25.69321439
X1	2.7260898	1.20	0.2483	2.27500488
X2	0.0472184	0.51	0.6199	0.09334856
X3	-2.5874439	-4.03	0.0010	0.64281819

a) What is the estimated regression model?
b) Test Ho: $\beta_1 = \beta_2 = \beta_3 = 0$.(完整列出檢定過程之每一步驟)
c) Test Ho: $\beta_3 = 0$. (完整列出檢定過程之每一步驟)
d) Find R^2, and interpret its meanings.
e) Set up a 90% confidence interval for β_3.

表值：$t_{.05}(16) = 1.746$; $t_{.05}(17) = 1.740$; $t_{.05}(18) = 1.734$; $t_{.05}(19) = 1.729$;
$t_{.025}(16) = 2.120$; $t_{.025}(17) = 2.110$; $t_{.025}(18) = 2.101$; $t_{.025}(19) = 2.093$;

本試題係兩面印刷

試題完