

機率論與微積分 試題

Statistics Department-Entrance Examination 88:

Calculus & Probability

Part I

(15%) 1. Evaluate the limit by recognizing as the value of an definite integral:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\sqrt{2in + i^2}(n+i)}{n^3}$$

(20%) 2. Find the value  $k$  such that  $\lim_{x \rightarrow 0} f(x)$  exists, if

$$f(x) = \begin{cases} (1 - e^x)^x & \text{if } x > 0, \\ \tan(kx)/x & \text{if } x < 0. \end{cases}$$

(15%) 3. Solve the double integral:

$$\int_0^{\sqrt{\ln 2}} \int_0^1 \frac{xye^{x^2}}{1+y^2} dy dx$$

Part II

(10%) 1. Suppose that a biased coin that lands on heads with probability  $p$  is flipped 10 times. Given that a total of 7 heads result, find the conditional probability that the first 4 outcomes are H, T, T, H (meaning that the first flip is heads, the second is tails, the third is tails and the last is heads).

(20%) 2. Suppose that a joint density function of two random variables  $X$  and  $Y$  is defined as

$$f(x, y) = e^{-(x+y)}, 0 < x < \infty, 0 < y < \infty.$$

Find the probability  $P\{X < Y\}$ ? Find the conditional expectation  $E(X|Y)$ ? Are  $X$  and  $Y$  independent? why?

(20%) 3. One manufacturing company produces two products A and B. Let  $p_a$  and  $p_b$  be the prices of A and B, respectively. Let  $q_a$  and  $q_b$  be the quantities of A and B, respectively. Their relationship are

$$p_a = 12 - 3q_a; p_b = 10 - q_b$$

and the cost function is  $C = q_a^2 + 2q_aq_b + q_b^2$ . Find the maximum profit?

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