銘傳大學 96 學年度研究所碩士班招生考試

應用統計資訊學系碩士班

第一節

機率論試題

（限用答案本作答）

1. (12%) Let $f_X(x) = ce^x I_{\{0,1\}}(x) + \frac{1}{8} I_{(0,2)}(x) + (\frac{1}{2})^x I_{\{2,3,\ldots\}}(x)$. Find c such that $f_X(x)$ is a probability density function.

2. (12%) Let X be a standard normally distributed random variable. Find the probability density function of $Y = X^2$.

3. (12%) Let $X \sim f_X(x) = \frac{1}{2\theta} I_{(-\theta,\theta)}(x)$. Find the probability density of $Y = X^2$.

4. (12%) A system will function as long as at least one of two components functions. Let X_i be the random variable denoting the lifetime of component i, $i = 1,2$. Assume that the X_i's are independent exponential random variables with parameter 1. Find the distribution of the lifetime of the system.

5. (24%) Let $f_{X,Y}(x,y) = e^{-\lambda(x+y)} I_{(0,\infty)}(x) I_{(0,\infty)}(y)$. Define $U = X + Y$ and $V = X/(X+Y)$. (a) Find the distribution of U. (b) Find $E(V)$.

6. (16%) Let $f_{X|Y}(x \mid y) = \frac{n!}{n!(n-x)!} y^x (1-y)^{n-x} I_{\{0,1,2,\ldots,n\}}(x)$ and

   $f_Y(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1} I_{(0,1)}(y)$. Find $E(X)$ and $Var(X)$.

7. (12%) Let $Var(X_i) = \sigma^2$ and $E(X_i) = \mu_i, i = 1,2$. Let ρ be the correlation coefficient of X_1 and X_2. Find the upper bound of $P[|(X_1 - \mu_1) + (X_2 - \mu_2)| \geq k\sigma]$.

試題完