

線性代數試題

(限用答案本作答)

一、(14%) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$

二、(14%) Use the "Gram-Schmidt algorithm" to convert the given basis B of V into an orthogonal basis that contains the vector (0,1,1). $V = \mathbb{R}^3$, $B = \{(0,1,1), (1,1,1), (1,-2,2)\}$

三、(14%) Find a linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ such that

$$T \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, T \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, T \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

四、(14%) Given $(I - 2A^T)^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$,

(a) find A (b) find $\det(A^9)$.

五、(14%) Let $\{u, v, w, z\}$ be linearly independent vectors. Which of the following are linearly independent? (選擇題)

- (a) $\{u-v, v-w, w-u\}$ (b) $\{u+v, v+w, w+u\}$ (c) $\{u-v, v-w, w-z, z-u\}$
 (d) $\{u+v, v+w, w+z, z+u\}$

六、(15%) Show that

$$\begin{vmatrix} 1+x_1 & x_2 & x_3 & \dots & x_n \\ x_1 & 1+x_2 & x_3 & \dots & x_n \\ x_1 & x_2 & 1+x_3 & \dots & x_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1 & x_2 & x_3 & \dots & 1+x_n \end{vmatrix} = 1+x_1+x_2+x_3+\dots+x_n$$

七、(15%) A matrix is skew-symmetric if $A^T = -A$. If A is an n by n skew-symmetric matrix, show that $\det(A) = (-1)^n \det(A)$.

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