管理科學

銘傳大學九十一學年度財務金融研究所碩士班招生考試
風險管理與統計資訊

第一節

微積分 試題

Part I. Multiple choice (40%)

1. Find the vertical asymptote(s), if any, for

$$f(x)=\frac{7x+4}{x^2-6x+5}$$

(a) x = $\frac{-4}{7}$ (b) no vertical asymptotes

(c) x = 5, x = 1, x = $\frac{-4}{7}$ (d) x = 5, x = 1

2. Find y' for y = y(x) defined implicitly by $4x + 7xy = 4e^{2y}$ and evaluate at x = 1, y = 0.

(a) $y'|_{(1,0)} = 11$ (b) $y'|_{(1,0)} = \frac{4}{7}$

(c) $y'|_{(1,0)} = 4$ (d) $y'|_{(1,0)} = \frac{-4}{3}$

3. The marginal revenue from the sale of compact discs is given by $R'(x) = 230 - 8x$ and R(0) = 0, where R(x) is the revenue in dollars. Find the price-demand equation.

(a) p = $\frac{230}{x} - 8$ (b) p = 230 – 4x

(c) p = 230 ln x – 8x (d) p = 230x – $4x^2$

4. Find the equilibrium price if the price-demand equation and the price-supply equation are

$P = D(x) = 60 - \frac{1}{10}x$ and $p = S(x) = 18 + \frac{1}{20}x$,

respectively. Use this value to find the consumers' surplus.

(a) \$3920 (b) \$980 (c) \$1960 (d) \$2940

5. Find

$$\int \frac{x}{(3+2x)(4+5x)}dx.$$

(a) $\frac{4}{35}\ln|4+5x| - \frac{3}{14}\ln|3+2x| + C$ (b) $\frac{3}{14}\ln|3+2x| - \frac{4}{35}\ln|4+5x| + C$

(c) $\frac{1}{7}\ln\left|\frac{4+5x}{3+2x}\right|+C$ (d) $\frac{1}{7}\ln\left|\frac{3+2x}{4+5x}\right|+C$

6. Use Lagrange multipliers to maximize f(x, y, z) = 28x + 14y + 28z subject to x2 + y2 + z2 = 441. Which of the following are true?
   (I) The maximum value of f(x, y, z) is less than or equal to 883
   (II) The maximum value of f(x, y, z) occurs when x = 14
   (III) The maximum value of f(x, y, z) occurs when y = 7
   (a) I only (b) I, II and III (c) I and III only (d) II and III only.
7. Find the interval of convergence of the Taylor series representation:

$$f(x)=\frac{7}{8-x}=\frac{7}{8}+\frac{7}{8^2}x+\ldots+\frac{7}{8^{n+1}}x^n+\ldots$$

(a) $|x|<\frac{7}{8}$ (b) $|x|<\frac{1}{8}$ (c) $|x|<\frac{8}{7}$ (d) $|x|<8$.

8. Evaluate $\int_{-\infty}^{8}e^{-5x}dx$ if it converges.

(a) $5e^{-40}$ (b) $\frac{e^{-40}}{5}$

(c) $-(\frac{e^{-40}}{5})$ (d) The improper integral diverges.

Part II. Free response (60%)

9. (20%) Find the following limits.

(a) $\lim_{x\to\infty}(\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n})$ (b) $\lim_{x\to 0}\frac{\int_0^x \sin t^2 dt}{x^3}$

(c) $\lim_{x\to 0}(1-3x)^{\frac{6}{x}}$ (d) $\lim_{x\to 1}\frac{\tan^{-1}x-\frac{\pi}{4}}{x-1}$

10. (15%) Compute the following definite integrals.

(a) $\int_0^3\left[\sqrt{9-x^2}-(3-x)\right]dx$

(b) $\int_2^3 \ln(x-1)dx$

(c) $\int_0^4\int_{\frac{x}{2}}^2 e^{y^2}dydx$

11. (10%) Let

$f(x) = 2x^7 + 4x^5 + x^3 + x - 3$.

(a) Show that the inverse function f^{-1} exists.

(b) Find $(f^{-1})'(5)$.

12. (15%) Graph

$f(x) = 3x^{\frac{2}{3}} - x$,

noting all critical points, extrema, inflection points, intervals where f(x) increases and decreases and asymptotes.

試題完